Bose - Einstein - condensation →

Bose - Einstein condensation is a quantum phenomenon in Bose gases in which a large number of bosons simultaneously occupy the ground state of a system.

* A state of matter in which a set of atoms or particles are chilled to such low energies that they 'condense' into a single quantum state (In which a statistical description of the positions of the atoms implies that they physically overlap each other and in effect from a single atom)

An assembly of bosons (i.e. indistinguishable elementary particles of zero or integral spin) is term as Bose - Einstein gas.

* Particles, like everything, have wave properties, such as wavelength. The trick is getting into a regime where the wave properties emerge.

* The wavelength (called De Broglie wavelength) of an atom is related to its temperature - the colder the atom, the longer the wavelength.

* At room temperature, atoms can be treated like billiard balls bouncing around.
* At low temperatures, the wavelengths become longer, and so the wave properties become relevant.

* For sufficiently low temperature, a few millionths of a degree above zero temperature the bosonic atoms effectively become overlapping waves that share the same phase. The atoms become a BEC, which has quantum mechanical behavior.

The BEC phenomenon was first predicted by Satyendra Bose and Albert Einstein in the 1920's, hence the name. BEC was first noted to exist in liquid helium.

Therefore expression for total number of particles of perfect Bose-Einstein gas

$$n = \frac{V}{h^3}(2\pi mKT)^{3/2} \sum_{r=1}^{\infty} \frac{A^r}{r^{3/2}} \qquad \left(\because n = \frac{V}{h^3}(2\pi mKT)^{3/2} f_1(\alpha) \right)$$

$$n = \frac{V}{h^3}(2\pi mKT)^{3/2} \left[A + \frac{A^2}{2^{3/2}} + \frac{A^3}{3^{3/2}} + \cdots \right] \quad \text{—(1)}$$

Then the behavior of the perfect gas departs farther and farther from that of the classical perfect gas due to the fact that the velocities of the particles are subject to quantum statistics and not to classical statistics.

The gas under this condition is said to be degenerate and the parameter A is called the degeneracy parameter.

For $A << 1$

$$n = \frac{V}{h^3}(2\pi mKT)^{3/2} A$$

The approximate value of A is from equation

$$A = \left(\frac{n}{V}\right)\frac{h^3}{(2\pi mKT)^{3/2}} \quad \text{—— (2)}$$

If the density of particles is increased and / or the temperature is decreased, the value of A increases.

$$\alpha = -\frac{\mu}{KT} \quad \text{decreases}$$

As the expression for A contains three variables -

1) m The mass of the particle
2) $\frac{n}{V}$ The particle density (the number of particles per unit volume)
3) T the absolute temperature of gas

obviously the criterion of degeneracy will be based on the magnitude of $\frac{\frac{n}{V}}{(mT)^{3/2}}$.

Hence the degree of degeneracy will be large when - temperature is low, particle idensity is large and the mass of each boson is small.

For ground state $A = 1$

For low energy values the maximum admissible value of A is 1 and consequently α can never be negative.

Thus for low energy values the limiting case of highest degeneracy in Bose-Einstein gas reaches when $A = 1$,

$\alpha = 0 \quad \therefore e^{\alpha} = \frac{1}{A}$

$$n = \frac{V}{h^3}(2\pi mKT)^{3/2}\left\{A + \frac{A^2}{2^{3/2}} + \frac{A^3}{3^{3/2}} + \cdots\right.$$

So the maximum value of particle density $\frac{n}{V}$ will be given by

$$\left[\frac{n}{V}\right]_{max} = \frac{(2\pi mKT)^{3/2}}{h^3}\left[1 + \frac{1^2}{2^{3/2}} + \frac{1^3}{3^{3/2}} + \cdots\right]$$

$$\left[\frac{n}{V}\right]_{max} = \frac{(2\pi mKT)^{3/2}}{h^3}(2.612) \quad \text{—— (3)}$$

Since equation (3) corresponds to the limiting case of the Bose Einstein degeneration.

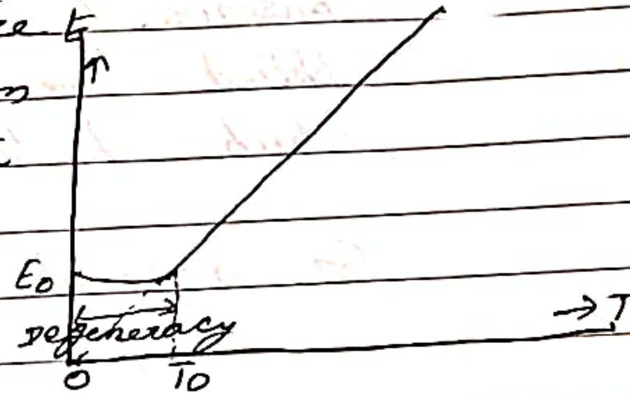
The fact that no value of n/V can be greater than that given by eqn (3) can be alternatively expressed in terms of the critical temperature T_0

$$\left[\frac{n}{V}\right]_{max} = \frac{(2\pi mK)^{3/2}}{h^3}(T_0)^{3/2}(2.612) \quad \text{—— (4)}$$

Thus the critical temperature T_0 is the lowest temperature for which a solution of equation ① is possible i.e. there is one solution of equation for $T < T_0$

T_0 is therefore the temperate at which the degeneracy of energy levels starts

* Below the critical temperature T_0 full line shows the relation between E and T of degenerate gas, while the dotted line through the origin that Non-degenerate gas.



* E_0 is termed as zero point energy which will be understood in the application of Fermi Dirac statistics

* The critical temperature T_0 is 5K for helium gas

Now, the number of particles lying between energy range energy E and E+dE is

$$n(E)dE = \frac{g(E)dE}{e^{(\alpha+\beta E_i)}-1} = \frac{g(E)dE}{e^{\left[\alpha+\frac{E_i}{KT}\right]}-1} \quad \text{—— (5)}$$

$$\because g(E) = \frac{4\pi m V}{h^3}\sqrt{2m}$$

NOTE → For ground state $E = E_0 = 0$, $g(E) = 0$ while actually it should be unity $g(0) = 1$ as there is one state at $E=0$ Therefore, the above distribution gives incorrect result for ground state, while this state is very important at low temperature

And at $E \neq 0$, $g(E) \neq 0$ therefore the above distribution eq^n (5) holds good.

consequently, the distribution (5) can still be applied for all states except ground state which should be treated separately.

For a single state

$$n_i = \frac{g_i}{e^{(\alpha+\beta E_i)}-1}$$

For ground state

$E_i = E_0, g_i = 1$ the number of particles in the ground state is $n \rightarrow n_0 = \frac{1}{e^{(\alpha+\beta 0)}-1} = \frac{1}{e^{\alpha}-1}$

Therefore, the total number "n" of particles for the degenerate case may be expressed as

$$n = n_0 + n' \quad \text{———} \quad (6)$$

$$n = n_0 + \int n(E)\,dE = n_0 + \int_0^{\alpha} \frac{4\pi m V}{h^3} \sqrt{2m} \frac{E^{1/2}\,dE}{e^{(\alpha+\frac{E}{KT})}-1} \quad \text{———} \quad (7)$$

$$n' = \frac{V}{h^3}(2\pi m K T)^{3/2}\left[A + \frac{A^2}{2^{3/2}} + \frac{A^3}{3^{3/2}} + \text{——}\right]$$

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$$n' = \frac{V}{h^3}(2\pi m K)^{3/2}(T)^{3/2}(2.612) \quad \text{—— (a)}$$

(for A = 1)

using eqn (4)

$$n = \frac{V(2\pi m K)^{3/2}}{h^3}(T_0)^{3/2}(2.612)$$

$$\frac{n}{(T_0)^{3/2}(2.612)} = \frac{V(2\pi m K)^{3/2}}{h^3}$$

Eqn. (a) can be become

$$n' = \frac{n}{(T_0)^{3/2}(2.612)}(T)^{3/2}(2.612) = n\frac{(T)^{3/2}}{(T_0)^{3/2}}$$

$$n' = n\left(\frac{T}{T_0}\right)^{3/2} \quad \text{—— (b)}$$

using eqn (b) in eqn (6)

$$n = n_0 + n' = n_0 + n\left(\frac{T}{T_0}\right)^{3/2}$$

$$n - n\left(\frac{T}{T_0}\right)^{3/2} = n_0$$

$$n_0 = n\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right] \quad \text{—— (8)}$$

for $T < T_0$

Therefore the particles n_0 must condense into the ground state.

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From eqn. (8), it is obvious that when the temperature of a Bose-Einstein gas is lowered below the critical temperature T_0 the number of particles in the ground state rapidly increases.

This rapid increase in the population of the ground state below the critical temperature T_0 for a Bose-Einstein gas is called the Bose-Einstein condensation.

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